ANALOG ELECTRONICS

PREPARED BY

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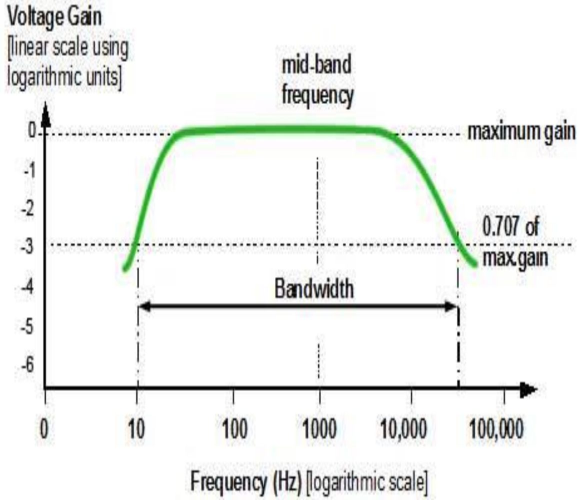
BJT FREQUENCY ANALYSIS

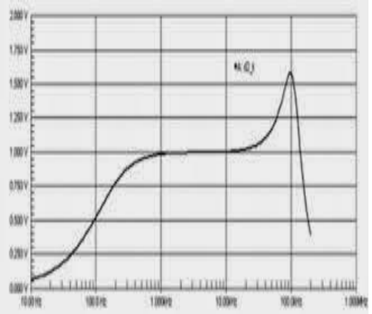
- **Frequency response of RC Coupled amplifier:**
- Fig. (1) shows the frequency response of RC coupled amplifier. The horizontal scale is a logarithmic scale to permit a plot extending from low to high frequency regions. The frequency range is divided into 3 regions.
- (i) Low frequency region.
- (ii) Mid frequency region.
- (iii) High frequency region.
- The drop in the gain at low frequencies

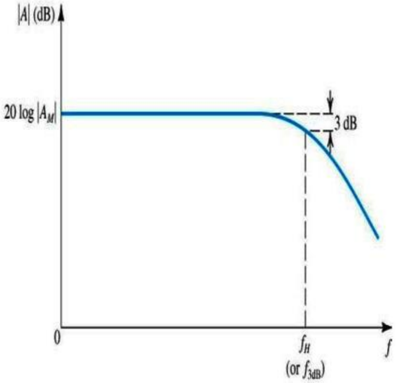
- FIG 1



- **Frequency response of transformer coupled amplifier:**
- Fig.(2) shows frequency response of transformer coupled amplifier. The magnetizing inductive reactance of the transformer winding is XL=2πfl. At low frequencies the gain drops due to small value of XL.
- At f=0 (DC) there is no change in flux in the core. As a result the secondary induced voltage or output voltage is zero and hence the gain. At high frequencies the gain drops due to stray capacitance between the turns of primary and secondary windings.

- FIG 3





- **Frequency response of direct coupled amplifier:**
- Fig. (3) shows frequency response of direct coupled amplifier. Since there are no coupling or bypass capacitors, there is no drop in gain at low frequencies. It has a flat response to the upper cut-off frequency. Gain drops at high frequencies due to device internal capacitances and the stray wiring capacitance

- **Half power frequencies and bandwidth:**
- The frequencies f1 and f2 at which the gain is 0.707 Avmid are called cut-off frequencies or corner frequencies or break frequencies. f1 is called the lower cut-off frequency and f2 is called the upper cut-off frequency.
- Bandwidth or pass band of the amplifier is

- BW= f2- f1----- (1)
- The output voltage in the mid band is |VO|=| Avmid | |Vi|
- Output power in the mid band is

$$P_{O(mid)} = \frac{|V_O|^2}{R_O}$$

$$= \frac{|Av_{mid}|^2 |V_i|^2}{R_O} \text{-----} (2)$$

The output voltage at cut-off frequencies is

$|V_O| = |0.707\mathrm{Av}_{mid}|\ |V_i|$

The output power at cut-off frequencies is

$$P_{O\,(\text{cut-off})} = \frac{|0.707\mathrm{Av}_{mid}|^2 |V_i|^2}{R_O}$$

$$= \frac{0.5\ |\mathrm{Av}_{mid}|^2 |V_i|^2}{R_O}$$

$= 0.5\ P_{O\,(\text{mid})}$ ----- (3)

NORMALIZED GAIN V/S FREQUENCY PLOT:

- The normalized gain is obtained by dividing the gain at each frequency by the mid band gain Avmid.
- Therefore normalized gain

$$= \frac{Av}{Av_{mid}} \text{ ----- } (4)$$

Fig. (4) shows the normalized gain V/s frequency plot for an RC coup

The normalized mid band gain is $\frac{Av_{mid}}{Av_{mid}}$ =1

The normalized gain at cut-off frequencies is $\frac{0.707Av_{mid}}{Av_{mid}}$ =0.707

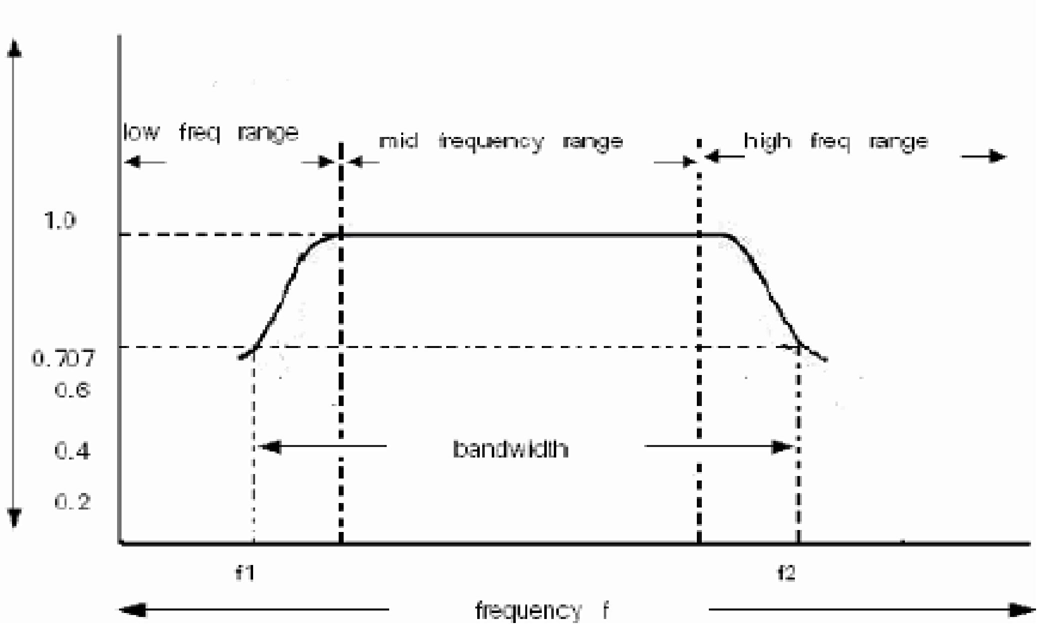


Fig. (4) Normalized gain V/s frequency plot

Normalized decibel gain is $\frac{A_V}{Av_{mid}}\Big|_{dB} = 20\log_{10}\left[\frac{A_V}{Av_{mid}}\right]$ -----(5)

Normalized decibel voltage gain at cut-off frequencies is

$$20 \log_{10}\left[\frac{0.707 A_V}{Av_{mid}}\right] = -3\text{dB}$$

Since normalized decibel voltage gain at cut-off frequencies is 3dB less tha normalized decibel mid band voltage gain. f_1 and f_2 are also called frequencies.

f_1 →lower 3dB frequency

f_2→upper 3dB frequency

Fig. (5) shows the plot of normalized dB voltage gain V/s frequency for a coupled amplifier.

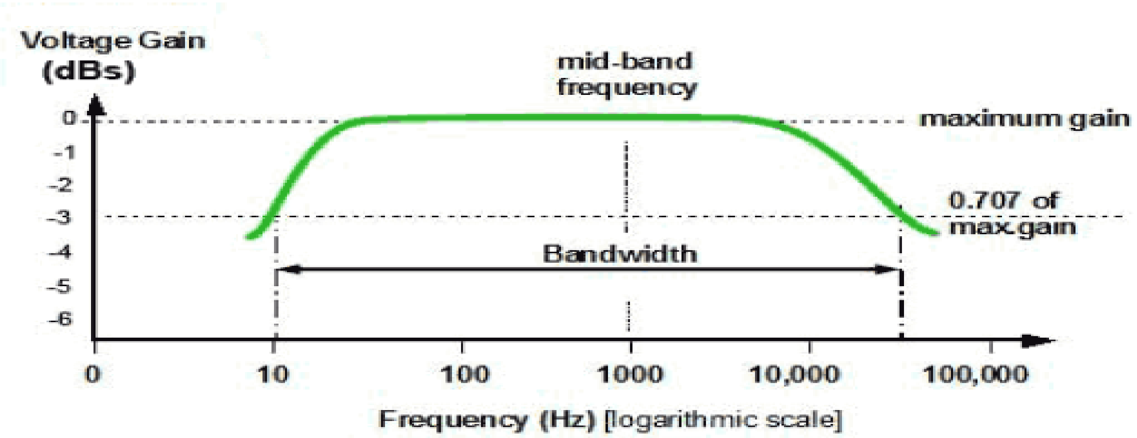


Fig. (5) Plot of normalized decibel voltage gain V/s frequency.

Low frequency Response of BJT amplifier:

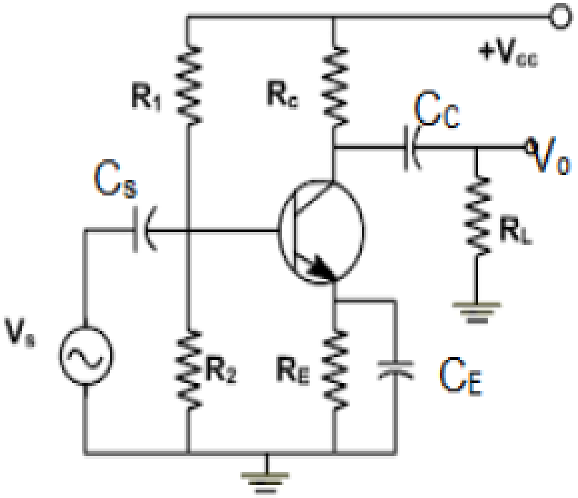
Fig. (13) shows the circuit of single stage BJT amplifier. The coupling capacitors C_S and C_C and bypass capacitor C_E determines the low frequency response.

Effect of C_S on low frequency response:

The input coupling capacitor C_S couples the source signal to BJT. First, we will neglect the effects of C_C and C_E i.e. they are treated as short circuits.

The AC equivalent circuit is obtained by reducing VCC to zero and C_C and C_E by their short circuit equivalent as shown in Fig. (14).

- The resistance of the transistor between base-emitter is hie



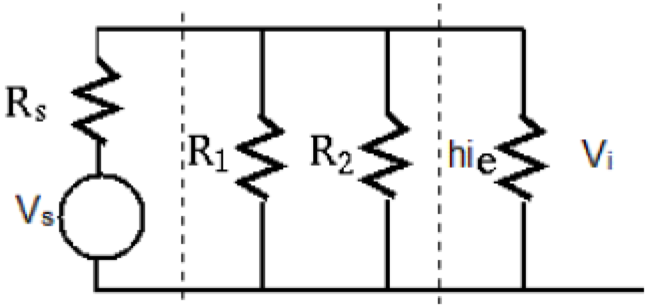


Fig. (15) Input AC equivalent

Let $R_i = R_1 \parallel R_2 \parallel h_{ie}$

Where $h_{ie} = \beta r_e$

Using voltage division rule in the circuit of the voltage applied to the amplifier is

$$V_i = \frac{V_S R_i}{(R_{S+} R_i) - f X c_S}$$

Where $Xc_S = \frac{1}{2\pi f C_S}$

$$V_i = \frac{V_S\left[\frac{R_i}{R_S+R_i}\right]}{1-j\left[\frac{Xc_S}{R_i+R_S}\right]}$$

$$V_i = \frac{|V_S|\left[\frac{R_i}{R_S+R_i}\right]}{\sqrt{1+j\left[\frac{Xc_S}{R_i+R_S}\right]^2}} \text{ --}$$

- In the mid frequency band, f is large. As a result

$$, Xc_S \rightarrow 0.$$

$$|V_i|_{mid} = \frac{|V_S| R_i}{(R_S + R_i)}$$

$$|V_i| = \frac{|Vi|mid}{\sqrt{1 + \left[\frac{Xc_S}{R_i + R_S}\right]^2}}$$

The lower 3dB cut-off occurs when $|V_i| = \frac{|Vi|mid}{\sqrt{2}} = 0.707\ |V_i|_{mid}$

$$0.707\,|V_i|_{mid} = \frac{|Vi|mid}{\sqrt{1+\left[\frac{Xc_S}{R_i+R_S}\right]^2}}$$

This condition is satisfies

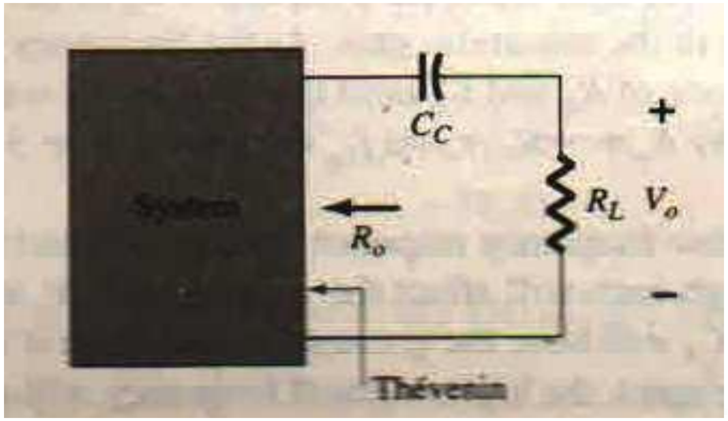
if $\frac{Xc_S}{R_i+R_S} = 1$ or $Xc_S = R_i + R_S$

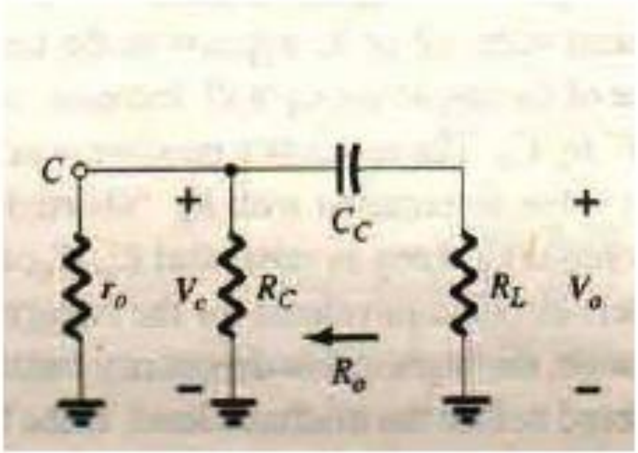
$$\frac{1}{2\pi f C_S} = R_i + R_S$$

Therefore f= $\frac{1}{2\pi(R_S+R_i)C_S}$

$$f_{Ls} = \frac{1}{2\pi(R_S+R_i)C_S}$$

- **I coupling capacitor CC on low frequency response :**
- The output coupling capacitor CC couples the output of the BJT to the load. The equivalent circuit on the output side by neglecting the effect of CS and CE by treating them as short circuits is as shown in Figure





Let $R_O = r_O \| R_C$

V_C= output voltage of BJT

V_O=load voltage

$$V_O = \frac{V_C R_L}{(R_O + R_L) - jXc_C} \;\text{--}$$

$$\text{Where } Xc_C = \frac{1}{2\pi f C_C}$$

$$V_O = \frac{V_C \left[\frac{R_L}{R_O + R_L}\right]}{1 - j\left[\frac{Xc_C}{R_O + R_L}\right]}$$

$$|V_O| = \frac{|V_C| \left[\frac{R_L}{R_O + R_L}\right]}{\sqrt{1 + j\left[\frac{Xc_C}{R_O + R_L}\right]^2}}$$

In the mid frequency band, $Xc_C \rightarrow 0$

Therefore $|V_O|_{mid} = \frac{|V_C| R_L}{(R_O + R_L)}$

$$|V_O| = \frac{|V_O|_{mid}}{\sqrt{1 + \left[\frac{Xc_C}{R_O + R_L}\right]^2}}$$

The lower 3dB cut-off occurs when $|V_O| = \frac{|V_O|_{mid}}{\sqrt{2}} = 0.707\ |V_O|_{mid}$

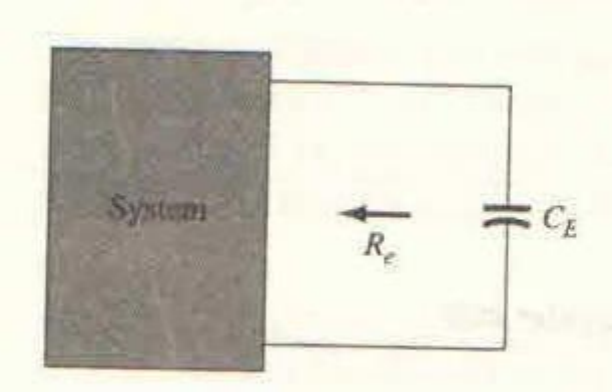
This is possible iff,

$\frac{Xc_C}{R_O + R_L} = 1$ or $Xc_C = R_O + R_L$

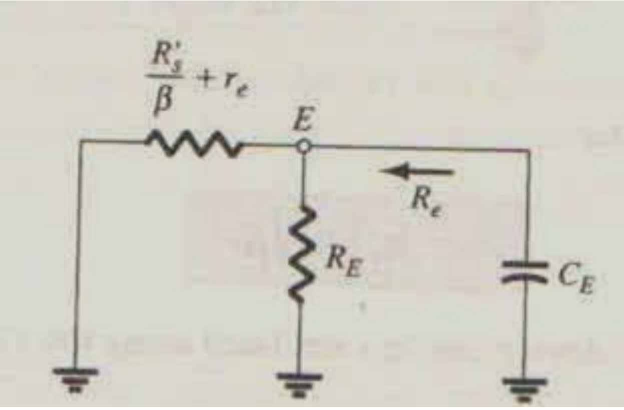
$$f= \frac{1}{2\pi(R_O+R_L)C_C} .$$

$$f_{Lc}= \frac{1}{2\pi(R_O+R_L)C_C} .$$

Effect of Emitter bypass capacitor CE on low frequency response : The equivalent circuit considering the effect of CE is as shown in Fig. Hence the effect of CS and CC are neglected.



- AC equivalent circuit using hybrid model



- ent resistance seen by Ce. To find Re, VS is reduced to 0.

- Let Z’= RS‖ R1 ‖ R2 .

$\acute{R}_S = \beta r_e$ is in base circuit. When it is transformed to emitter circuit, it is divided by β. Therefore $I_E \approx I_C = \beta I_B$.

$$R_e = R_E \parallel \frac{\acute{R}_S}{\beta} + r_e$$

$$f_{L_E} = \frac{1}{2\pi R_e C_E}.$$

Effect of C_E on voltage gain:

The mid band voltage gain of amplifier of Fig. (13) without C_E is given by,

$$A_{V_{mid}} = -\frac{R_O||R_L}{r_e+R_E} \text{ ----- } (38)$$

Where $R_O = R_C||r_O$

If C_E is connected in parallel with R_E , then voltage gain becomes a function of frequency. The voltage gain at any frequency is

$$A_V = -\frac{R_O||R_L}{r_e + R_E||X_{C_E}} \text{ ----- (39)}$$

$$\text{Where } X_{C_E} = \frac{1}{2\pi f C_E} \text{ ----- (40)}$$

As the frequency increases:

(i) X_{C_E} decreases.
(ii) $R_E || X_{C_E}$ decreases.
(iii) A_V increases in magnitude.

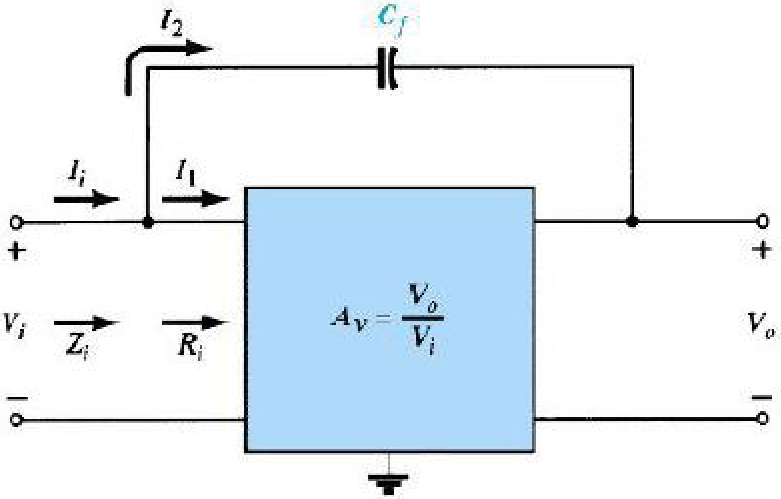
As the frequency approaches the mid band value

(i) X_{C_E} approaches zero.
(ii) $R_E || X_{C_E}$ approaches zero. (i.e. R_E is shorted out)
(iii) A_V approaches maximum value or mid band value.

$$A_{V_{mid}} = -\frac{R_O||R_L}{r_e} \text{ ----- (41)}$$

Miller Effect Capacitance:

Fig. (19) shows an inverting amplifier with a capacitance C_f between the input and output nodes. WKT, A_V is –Ve for inverting amplifier since V_O and V_i are 180^0 out of phase. Using Millers theorem we can find the loading effect of C_f on the input and output circuits of the amplifier.



To find Miller-Input Capacitance (C_{mi}) :

From Fig. (19),

$$R_i = \frac{V_i}{I_1} \rightarrow I_1 = \frac{V_i}{R_i}$$

$$Z_i = \frac{V_i}{I_i} \rightarrow I_i = \frac{V_i}{Z_i}$$

Apply KCL at input node A,

$$I_i = I_1 + I_2 \text{ ----- } (42)$$

From Fig. (23),

$$I_2 = \frac{V_i - V_o}{X_{C_f}}$$

But $V_o = A_V V_i$

Therefore $I_2 = \frac{V_i - A_V V_i}{X_{C_f}} = \frac{V_i[1 - A_V]}{X_{C_f}}$

Substitute for I_i , I_1 and I_2 in Eq. (42), we get

$$\frac{V_i}{Z_i} = \frac{V_i}{R_i} + \frac{V_i[1-A_V]}{X_{C_f}}$$

Eliminating V_i throught,

$$\frac{1}{Z_i} = \frac{1}{R_i} + \frac{[1-A_V]}{X_{C_f}} = \frac{1}{R_i} + \frac{1}{\frac{X_{C_f}}{1-A_V}}$$

Let $X_{C_{mi}} = \frac{X_{C_f}}{1-A_V}$ ----- (43)

$$\frac{1}{Z_i} = \frac{1}{R_i} + \frac{1}{X_{C_{mi}}} \text{ ----- (44)}$$

But $X_{C_f} = \frac{1}{2\pi f C_f}$

$$X_{C_{mi}} = \frac{1}{[1-A_V]2\pi f C_f} \text{ ----- (45)}$$

Where $C_{mi} = [1 - A_V]C_f$ = miller input capacitance

To find Miller output capacitance (C_{mo}) :

From Fig. (19),

$R_O = \frac{V_o}{V_i}$ ➔ $I_1 = \frac{V_o}{R_O}$

$Z_O = \frac{V_o}{I_O}$ ➔ $I_O = \frac{V_o}{Z_O}$

Apply KCL at node B,

$I_0 = I_1 + I_2$ ----- (47)

Statement of Millar's theorem:

A capacitance C_f connected between the input and output nodes of an inverting amplifier can be replaced by

(i) Miller input capacitance, $C_{m_i} = = [1 - A_V]C_f$ connected between input node and ground.

(ii) Miller output capacitance, $C_{m_o} = \left[1 - \frac{1}{A_V}\right] C_f$ connected between output node and ground.

For an non-inverting amplifier A_V is positive. In order to obtain positive values for C_{m_i} and C_{m_o}. Eq. (46) and Eq. (49) should be modified as follows

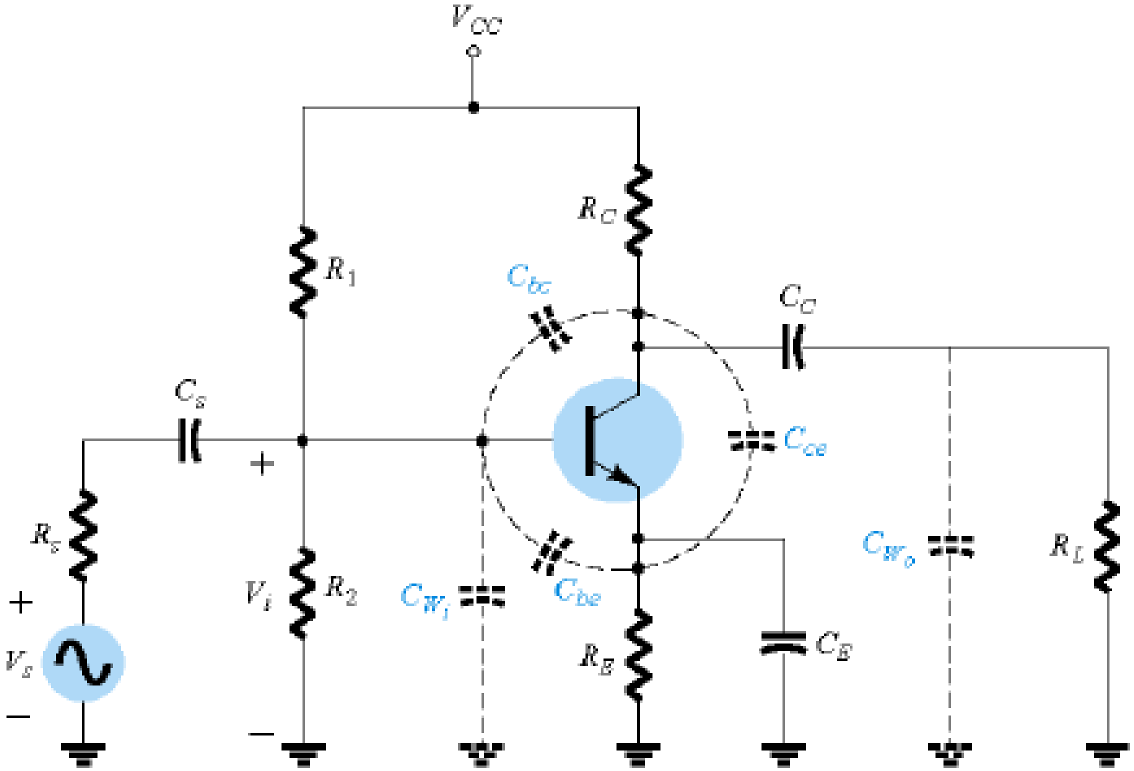
$C_{m_i} = = [1 - A_V]C_f$ ----- (50)

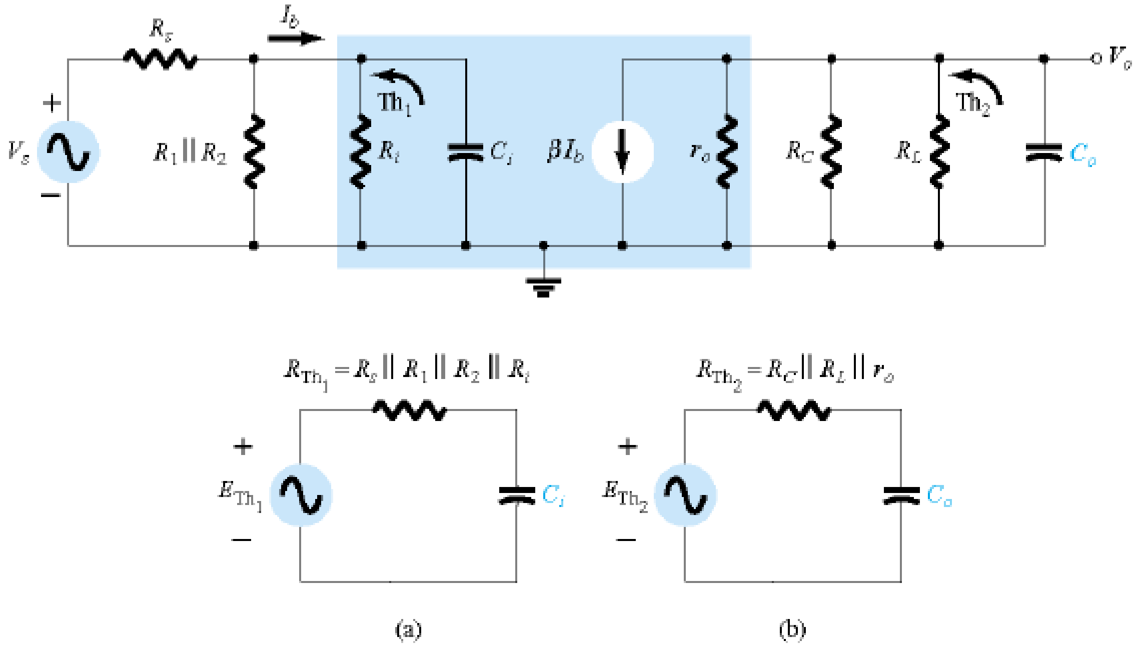
$C_{m_o} = \left[1 + \frac{1}{A_V}\right] C_f$ ----- (51)

- **High Frequency Response of BJT amplifier:**
- In the high frequency response of BJT amplifier, the upper 3dB cut-off point is defined by the following factors.

- (i) The network capacitance which includes the parasitic capacitances of the transistor and the wiring capacitances.

- (ii) The frequency dependence of short circuit current gain


V_{CC}
R_C
R_1
C_{bc}
C_C
C_s
+
C_{ce}
R_s
C_{W_o}
R_L
V_i
R_2
C_{W_i}
C_{be}
+
V_s
−
R_E
C_E
−


R_s
I_b
V_o
$+$
V_s
$-$
$R_1 \| R_2$
Th_1
R_i
C_i
βI_b
r_o
R_C
R_L
Th_2
C_o
$R_{\mathrm{Th}_1} = R_s \| R_1 \| R_2 \| R_i$
$+$
E_{Th_1}
$-$
C_i
$R_{\mathrm{Th}_2} = R_C \| R_L \| r_o$
$+$
E_{Th_2}
$-$
C_o
(a)
(b)

Using Miller's theorem, the transit capacitance, C_{bc} can be replaced by two capacitances; C_{mi} at the input and C_{mo} at output.

The total capacitance C_i is the sum of C_{mi}, C_{be} and C_{wi}.

i.e. $C_i = C_{mi} + C_{be} + C_{wi}$ ----- (52)

where $C_{mi} = [1 - A_V]C_{bc}$ ----- (53)

The total output capacitance is the sum of C_{mo}, C_{ce} and C_{wo}.

i.e. $C_0 = C_{wo} + C_{ce} + C_{mo}$ ----- (54)

where $C_{m_o} = \left[1 + \frac{1}{A_V}\right] C_f$

Upper cut-off frequency due to C_i:

Apply voltage division rule to circuit of Fig. (22),

$$E_{Thi} = V_s \left[\frac{R_1 \| R_2 \| \acute{R}_{\iota}}{R_S + R_1 \| R_2 \| \acute{R}_{\iota}}\right] \text{ ----- } (56)$$

From circuit in Fig. (21);

$$R_{Thi} = R_S + R_1 \| R_2 \| \acute{R}_{\iota} \text{ ----- } (57)$$

Where $\acute{R}_{\iota} = \beta r_e$

From Fig. (29) (b), Apply V_g division rule,

$$|V_i| = |E_{Thi}| \left[\frac{X_{C_i}}{\sqrt{(R_{Thi})^2 + (X_{Ci})^2}}\right]$$

$$|V_i| = |E_{Thi}| \frac{|E_{Thi}|}{\sqrt{1 + \left(\frac{R_{Thi}}{X_{Ci}}\right)^2}} \text{ ----- } (58)$$

Where $X_{Ci} = \frac{1}{2\pi f C_i}$ ----- (59)

In the mid band, the effect of C_i is negligible. As a result, X_{Ci} can be treated as open circuit i.e. $X_{Ci} = \infty$.

Therefore $|V_i|_{mid} \approx |E_{Thi}|$

At high frequencies, C_i cannot be neglected with increase in f, X_{Ci} decreases, $\frac{R_{Thi}}{X_{Ci}}$ increases, $|V_i|$ decreases and hence the voltage gain decreases.

3dB cut-off occurs at a frequency at which

$$|V_i| = \frac{|\mathrm{Vi}|\mathrm{mid}}{\sqrt{2}} = \frac{|E_{Thi}|}{\sqrt{2}}$$

From (58), this condition occurs, when

$$R_{Thi} = X_{Ci}$$

$$R_{Thi} = \frac{1}{2\pi f C_i}$$

Or $f = f_{\mathrm{Hi}} = \frac{1}{2\pi R_{Thi} C_i}$

$$=\left[\frac{f}{f_{Hi}}\right]$$

Therefore Eq. (58) becomes,

$$|V_i| = \frac{|E_{Thi}|}{\sqrt{1+\left(\frac{f}{f_{Hi}}\right)^2}} \text{ ----- } (62)$$

Thus, due to C_i, V_g gain decreases at the rate of 20dB/decade.

Upper cut-off frequency due to output capacitance C_O :

Consider the output circuit of Fig.(22) which is shown in Fig. (23).

βI_b, r_o and $R_C||R_L$ is connected to voltage source as shown in Fig. (22).

$$R_{Tho} = r_o \;||R_C||R_L \text{ ----- } (63)$$

$$E_{Tho} = [-\beta I_b]\;[r_o \;||R_C||R_L] \text{ ----- } (64)$$

Using the same procedure as listed above, we have

$$|V_o| = \frac{|E_{Tho}|}{\sqrt{1+\left(\frac{R_{Tho}}{X_{Co}}\right)^2}} \text{ ----- } (65)$$

Where $X_{Co} = \frac{1}{2\pi f C_o}$ ----- (66)

Combined effect of C_i and C_O on high frequency response:

(i) The input capacitance C_i, defines upper cut-off frequency f_{Hi}.

(ii) The output capacitance C_o, defines another upper cut-off frequency f_{Ho}.

(iii) The lowest of these 2 frequencies will be taken as overall upper cut-off frequency.

(iv) If the variation of h_{fe} with frequency is considered then the actual cut-off frequency may be lower than f_{Hi} or f_{Ho}.